

Radiometry and the Friis Transmission Equation

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Abstract—The Friis transmission equation is developed from principles of optical radiometry and scalar diffraction. This development provides students a link between concepts learned in antennas or communication courses and optical engineering and photonics courses. The radiometric approach helps students understand the wavelength dependence of the Friis transmission equation better than they do with a derivation based on dipole equivalent area.

Index Terms—Friis, Propagation, Radiometry, Diffraction.

I. INTRODUCTION

STUDENTS can benefit from being taught connections between material from courses on photonics, antennas, and communication systems. One promising opportunity for such a connection arises with the Friis transmission equation [1]. When taught the usual textbook derivation of this equation, students usually do not understand the physical source of the wavelength-squared term that expresses the effect of diffraction. They quickly learn to use the equation with ease, but without a clear conceptual understanding of why the equation takes the form that it does. In antenna courses this is compounded by the application to all antennas of the equation for effective area, derived from an infinitesimal dipole [2,3]. Introductory communications texts typically present the Friis equation with a statement of the relationship between antenna effective area and wavelength without derivation [4,5]. Photonics texts either skip the subject [6] or discuss some optical radiometry without mentioning the Friis equation [7].

It is becoming increasingly common for electrical engineering and physics students to take courses on photonics [8-10]. Many seniors and graduate students in a first course on antennas have been exposed to enough basic photonics and optical engineering principles that a development of the Friis transmission equation based on optical radiometry and scalar diffraction is intuitively appealing and leads to improved understanding of both the Friis equation and the radiometric principles. Even when students have no prior experience with radiometry, they find this development more physically intuitive than the dipole-equivalent-area approach that is

usually given in antenna and communications textbooks.

The development described here relies on the geometrical optics propagation of radiance [power per (area \times solid angle)] through free space with beam spreading introduced via simple scalar diffraction by the antenna aperture. This approach still relies on equivalent antenna areas, but in a more direct manner than the unique dipole-equivalent-area case. The only concept used in the radiometric development that a few students in our antenna class have not learned previously is scalar diffraction theory. However, these students still benefit from this approach because in a review of basic diffraction principles they learn that the wavelength-squared term in the Friis equation comes from the wave being diffracted into an angle that is proportional to wavelength and inversely proportional to the effective antenna dimension.

II. FRIIS EQUATION FROM DIPOLE EQUIVALENT AREA

Despite the simplicity of the concept of antenna equivalent area, the dipole derivation found in several popular antenna texts loses some of the important physical meaning in concatenated discussions of infinitesimal dipole radiation resistance, directivity, and the relationship between directivity and effective area. This derivation is reviewed here in a manner reasonably consistent with the notation and terminology in common texts [2,3].

By assuming a constant current on a lossless, impedance-matched infinitesimal dipole, a radiation integral can be solved to obtain the magnetic vector potential, the curl of which gives the dipole magnetic field. This field in turn produces a dipole electric field through Ampere's Law. These fields can be used to compute the time-average power density:

$$\vec{W} = \frac{1}{2} \vec{E} \times \vec{H}^* = \hat{r}W_r + \hat{\theta}W_\theta, \quad (1)$$

where \vec{E} is the electric field vector, \vec{H}^* is the complex conjugate of the magnetic field vector, \hat{r} is the radially directed unit vector, $\hat{\theta}$ is the azimuthally directed unit vector, and W_r and W_θ are the radial and azimuthal components of complex power density [W/m²]. The radial component of power density is integrated over a closed sphere to obtain the total power flowing in the radial direction, P_{rad} , which is equated to the power dissipated in the radiation resistance R_r :

$$P_{\text{rad}} = \eta \frac{\pi}{3} \left| \frac{I_0 l}{\lambda} \right|^2 = \frac{1}{2} |I_0|^2 R_r. \quad (2)$$

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In eq. (1) η is the impedance of free space ($\approx 120\pi$), I_o is the dipole current amplitude, l is the dipole length, and λ is the wavelength. Eq. (2) is solved for R_r to obtain

$$R_r = \eta \left(\frac{2\pi}{3} \right) \left(\frac{l}{\lambda} \right)^2 = 80\pi^2 \left(\frac{l}{\lambda} \right)^2, \quad (3)$$

the radiation resistance of a lossless, matched infinitesimal dipole ($l \ll \lambda$) in free space.

The effective area of the antenna A_e is defined as the area which multiplied by the incident power density W_i gives the power delivered at the terminals of the antenna P_T so that

$$A_e = \frac{P_T}{W_i} = \frac{|I_T|^2 R_r / 2}{W_i}. \quad (4)$$

Assuming the conjugate-matching conditions of maximum power transfer into an antenna with radiation resistance R_r and loss resistance R_L , the maximum effective area is shown to be

$$A_{em} = \frac{|V_T|^2}{8W_i} \left(\frac{1}{R_r + R_L} \right). \quad (5)$$

For a lossless antenna ($R_L = 0$) in free space and an incident uniform plane wave with electric field E and power density $W_i = E^2/2\eta$, we can use eq. (3) in (5) to write

$$A_{em} = \frac{(El)^2}{8 \left(\frac{E^2}{2\eta} \right) \left(\frac{80\pi^2 l^2}{\lambda^2} \right)}. \quad (6)$$

This can be algebraically reduced, using $\eta = 120\pi$, to

$$A_{em} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2. \quad (7)$$

The important wavelength-squared dependence of the Friis transmission equation becomes apparent at this point in this derivation, but the physical meaning is often lost for students trying to follow the lengthy process. Having the wavelength-squared term come from the radiation resistance equation into the effective area equation does not necessarily provide an intuitive insight into its meaning.

At this point we take a short detour to find the directivity of the infinitesimal dipole. The previously derived dipole fields are used, along with knowledge that the product of a power density and range squared gives the radiation intensity U [W/sr], to write

$$U = r^2 W_{av} = \frac{r^2}{2\eta} |E_\theta|^2 = \frac{\eta}{2} \left(\frac{I_o l}{2\lambda} \right)^2 \sin^2 \theta \quad (8)$$

The maximum radiation intensity U_{\max} occurs at $\theta = \pi/2$:

$$U_{\max} = \frac{\eta}{2} \left(\frac{I_o l}{2\lambda} \right)^2 \quad (9)$$

from which the maximum directivity is found as

$$D_o = \frac{U_{\max}}{U} = 4\pi \frac{U_{\max}}{P_{rad}} = 4\pi \frac{\eta \left(\frac{I_o l}{2\lambda} \right)^2}{\eta \frac{\pi}{3} \left| \frac{I_o l}{\lambda} \right|^2} = \frac{3}{2} = 1.5 \quad (10)$$

The derivation continues with the development of an equation that relates the maximum effective area to the maximum directivity of the infinitesimal dipole. This begins by noting that a directive transmitting antenna radiates a power density at range R equal to the isotropic power density multiplied by the antenna directivity:

$$W_t = W_o D_t = \frac{P_t D_t}{4\pi R^2}. \quad (11)$$

The received power is therefore

$$P_r = W_t A_r = \frac{P_t D_o A_r}{4\pi R^2}, \quad (12)$$

where A_r is the effective area of the receiving antenna.

If we add subscripts to indicate maximum values of D and A , this equation can be rearranged to the form

$$D_{or} A_{tm} = \frac{P_r}{P_t} (4\pi R^2), \quad (13)$$

which by reciprocity can also be written as

$$D_{ot} A_{rm} = \frac{P_r}{P_t} (4\pi R^2). \quad (14)$$

By equating (13) and (14), we arrive at

$$\frac{D_{ot}}{A_{tm}} = \frac{D_{or}}{A_{rm}}, \quad (15)$$

which says that increasing the directivity of an antenna results in a proportional increase of effective area. An isotropic transmitting antenna would have $D_{ot} = 1$, so its maximum effective area is

$$A_{tm} = \frac{A_{rm}}{D_{or}} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi}. \quad (16)$$

Therefore, the maximum effective area of an infinitesimal dipole used as a receiving antenna is

$$A_{rm} = D_{or} A_{tm} = D_{or} \left(\frac{\lambda^2}{4\pi} \right). \quad (17)$$

The textbooks at this point state that this relationship is true for all antennas, even though it was derived specifically for an infinitesimal dipole. This causes students some discomfort and continues to add layers that obscure the physical meaning of the wavelength-squared term.

Once the applicability of eq. (17) to all antennas has been accepted, this equation can be used in eq. (12) to write an expression for the power received by a nonisotropic receiving antenna (with multiplicative efficiency terms e_r and e_i):

$$P_r = A_r W_t = e_r e_i D_r \left(\frac{\lambda^2}{4\pi} \right) \left(\frac{P_t D_t}{4\pi R^2} \right). \quad (18)$$

From eq. (18) we can write the Friis transmission equation in terms of a ratio of received to transmitted power, using either antenna directivity or gain G :

$$\frac{P_r}{P_t} = e_r e_t \left(\frac{\lambda}{4\pi R} \right)^2 D_r D_t = \left(\frac{\lambda}{4\pi R} \right)^2 G_{or} G_{ot} \quad (19)$$

Thus we arrive at the desired equation, having seen along the way several important antenna terms and relationships, but having also perhaps lost sight of some of the physical significance of the free-space spreading term in parentheses. At this point it can be instructive to take the students through a brief summary of optical radiometry and show them how a simple radiometric calculation, coupled with a bit of scalar diffraction theory, also produces the same result.

III. REVIEW OF RADIOMETRIC CONCEPTS

The radiometric development of the Friis transmission equation relies on the concepts of radiance and throughput, both central to optical radiometry [7,11-13]. Radiance is the amount of power incident on (or emitted from) a surface per unit area from (or into) a given solid angle and has units of W/(m² sr). Suggestive of its actual meaning, radiance often is called brightness in the microwave radiometry and radio astronomy communities [14,15]. The concepts of radiance and the related radiometric quantities can be introduced and used in courses ranging from antennas to photonics and optical design because they provide a consistent framework for calculating power transmitted from or received by an antenna or optical detector. Despite the confusion too often found in radiometric discussions, only the five quantities listed in Table 1 are necessary for power flow calculations. Of these quantities, radiance is the most fundamental because it is invariant in a lossless system. Similarly, conservation of energy requires that the product of area and solid angle is invariant, a quantity referred to in the optical engineering community as throughput (antenna texts provide an added benefit to this optical concept by showing that all diffraction-limited systems have throughput equal to the wavelength squared [2,3]).

Table 1. Radiometric quantities and antenna equivalents

Quantity	Symbol	Units	Antenna equivalent
Power	P	W	Power, P
Irradiance	E	W / m ² Incident	Power density, W
Exitance	M	W / m ² leaving surface	Power density, W
Intensity	I	W / sr	Radiation intensity, U
Radiance	L	W / (m ² sr)	Brightness, B (in radio astronomy)

Whether or not the classroom discussion extends to a full coverage of all the radiometric quantities in Table 1, the most basic of radiometric principles can be taught and used to

derive the Friis transmission equation, the radar range equation, and other similar equations. One of the most important of these basic principles is that the throughput is the product of an area and a solid angle that always opens away from the area, as shown in Fig. 1. A short discussion of solid angles and steradians is usually required at this point. Students often need to be reminded that the steradian is a dimensionless unit of solid angle that can be understood in the same way as a more common planar angle. A diagram like Fig. 2 reminds students that an angle in radians is defined as the ratio of arc length and the radial distance from the arc to the vertex of the angle (S/R) and, similarly, a solid angle in steradians is defined as the ratio of a spherical area to the square of the radial distance to the vertex (A/R^2). In the far field, which is a requirement of the Friis equation, we can use a small-angle approximation and replace the spherical surface area in a solid angle with a flat cross-sectional area (such as the effective area of an antenna). In other words, a far-field solid angle subtended by an object can be calculated simply as the ratio of the object's cross-sectional area divided by the square of the distance to that area:

$$\Omega = \frac{A}{R^2} \quad (20)$$

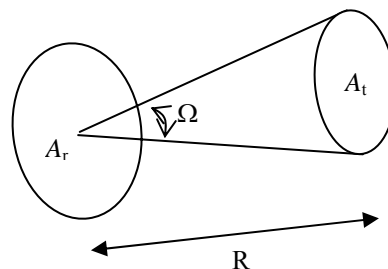


Fig. 1. Geometry for radiometric calculations, showing that a solid angle always opens away from the area (A_r) in which incident power is being calculated.

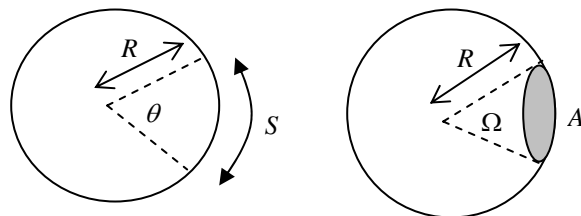


Fig. 2. In plane geometry an angle θ is defined as the ratio of arclength S over circle radius R (left); in spherical geometry a solid angle Ω is the ratio of area A on the sphere over the square of the radius R^2 .

This definition of a solid angle can be used to show that the power collected by a receiver area A_r from a solid angle Ω is given by the product of the source radiance L and the

geometric throughput $A_r\Omega$:

$$P = LA_r\Omega_t \quad (21)$$

For example, in a Friis transmission scenario, the area A_r represents the effective area of a receiving antenna and Ω_t (given by A_t/R^2) is the solid angle subtended by the transmitting antenna with effective area A_t and located at a distance R from the receiver. Because the throughput is equal to the product of the two areas divided by the square of the distance between them, regardless of which is the receiver and which is the transmitter, the throughput is an invariant of the system (combination of two antennas or optical apertures at a fixed distance). Consequently, conservation of power ($P = LA\Omega$) in a lossless medium requires that the radiance is invariant at all points in the system.

In a course on antennas it is important to emphasize that the receiver solid angle used in the Friis transmission equation is not the “beam solid angle” except in the unlikely case of the transmit antenna beam exactly filling the receiver field of view at distance R ; rather, the solid angle is simply that subtended by one antenna at the other, a solid angle that usually is smaller than the antenna beam solid angle (see discussion of eq. 32 in section IV).

IV. FRIIS EQUATION FROM RADIOMETRY AND DIFFRACTION

To develop the Friis transmission equation from optical radiometry and scalar diffraction theory, we begin with a common antenna calculation to incorporate the unique antenna concept of directivity into radiometry. The directivity of any antenna is given by the ratio of its solid angle Ω to an isotropic solid angle ($\Omega_o = 4\pi$ sr), which for power P can be written as

$$D = \frac{U}{U_o} = \frac{P}{\Omega} \frac{4\pi}{P} = \frac{4\pi}{\Omega} \quad (22)$$

This concept of solid angle ratio is used often in optical radiometry, but is not usually referred to as directivity, a useful term that probably should be incorporated by the optics community. In the far field, where the Friis equation is valid, the solid angle of a transmitted beam can be written as the cross-sectional beam area A_b divided by the range squared:

$$\Omega_t = \frac{A_b}{R^2} \quad (23)$$

Using eq. (23) in eq. (22) we obtain

$$D_t = \frac{4\pi R^2}{A_b} \quad (24)$$

It is here that diffraction behavior must be invoked to get the wavelength into the equation. Some students have not studied scalar diffraction theory, but many have in courses on photonics and Fourier optics [6,16]. Even for those students who have not encountered this subject before, it is easy to explain that diffraction causes a beam of electromagnetic energy to spread into an angle θ that is proportional to the ratio of wavelength λ over the dimension d of the aperture or

obstruction causing the diffraction:

$$\theta \cong \frac{\lambda}{d} \quad (25)$$

This is exactly true if d represents the dimension of a square aperture, but if d is the radius of an aperture of circular cross section eq. (25) is multiplied by 1.22 to obtain the Airy disk radius [16]. This is a detail that can be discussed or not, depending on student background and instructor interest. At range R sufficiently large that the beam dimension is much greater than the original aperture area, eq. (25) and a small-angle approximation results in the following estimate for the transverse dimension of the beam:

$$y \cong R\theta = \frac{R\lambda}{d}, \quad (26)$$

from which the beam area can be approximated as

$$A_b \cong y^2 = \frac{R^2 \lambda^2}{d^2} = \frac{R^2 \lambda^2}{A_t} \quad (27)$$

where A_t is the effective area of the transmit antenna. We can now substitute eq. (27) into eq. (24) to obtain an expression for the transmit antenna directivity,

$$D_t = \frac{4\pi R^2 A_t}{\lambda^2 R^2} = \frac{4\pi A_t}{\lambda^2}, \quad (28)$$

which in turn is solved to obtain the previous relationship (eq. 17) expressing the antenna effective area in terms of directivity:

$$A_t = \left(\frac{\lambda^2}{4\pi} \right) D_t \quad (29)$$

In this development students see directly that the wavelength-squared term expresses the diffraction-induced beam spreading of an electromagnetic wave launched from a finite aperture. In fact, eqs. (22) and (29) together reproduce the traditional antenna result,

$$A\Omega = \lambda^2, \quad (30)$$

which radiometrically says that the beam throughput (product of aperture area and beam solid angle) is equal to the wavelength squared. In both the antenna and radiometric discussions of the Friis equation this equation plays a central role because it demonstrates that as the wavelength increases, either the aperture (antenna) area must increase proportionally or the beam will spread into a proportionally larger solid angle. This point was arrived at using only simple geometric radiometry and a basic result of scalar diffraction theory, without employing radiation resistances, equivalent circuits, power-transfer relations, or anything unique to an infinitesimal dipole. The path through those quantities is a useful one, especially for electrical engineering students who are trained to use such concepts, but presenting this method in addition to the other enhances understanding.

The Friis equation derivation continues with the recognition that, in radiometric terminology, the transmit antenna emits a radiance $[W/(m^2 \text{ sr})]$ given by the ratio of transmitted power

to the transmitter throughput (product of effective transmit area and beam solid angle).

$$L_t = \frac{P_t}{A_t \Omega_b} = \frac{P_t R^2}{A_t A_b}, \quad (31)$$

where the last step invoked the far-field approximation of the solid angle. A very interesting result is found when eq. (27) is substituted in for the beam area A_b ,

$$L_t = \frac{P_t R^2}{A_t} \frac{A_t}{R^2 \lambda^2} = \frac{P_t}{\lambda^2}, \quad (32)$$

namely that the transmitted radiance can be written simply as the ratio of transmitted power to the wavelength squared. This again is a result of the throughput equality expressed in eq. (30) for a diffraction-limited system. This equality is another relatively common antenna concept not widely used in optics, which probably should be used more often in radiometric discussions of diffraction-limited optical systems.

Radiance propagates unchanged in a lossless medium, so all that is required to find the received power is to multiply the radiance by the appropriate receiver throughput. Here is where at least one student will always make the mistake of multiplying by the effective receiver area (correct) and the receive-antenna beam solid angle (incorrect). Doing this is simply equivalent to multiplying eq. (32) by λ^2 , which results in the incorrect answer that all the transmitted power is collected by the receiver. Actually, this would be the correct answer in the special (but unusual) case where the receiver beam exactly fills the area of the transmit antenna. As indicated in Fig. 1, the correct solid angle to use in the received-power calculation is the solid angle subtended by the transmit antenna at the receiver (A_t/R^2). The correct area is the effective area of the receive antenna. Using this product (not equal to λ^2) results in the following:

$$P_r = L_t A_r \Omega_{tr} = \frac{P_t}{\lambda^2} \frac{A_r A_t}{R^2}. \quad (33)$$

The final step is to substitute in the appropriate forms of eq. (29) to convert the effective receiver and transmitter areas into quantities involving directivities:

$$\frac{P_r}{P_t} = \frac{1}{\lambda^2} \frac{\lambda^2 D_t}{4\pi} \frac{\lambda^2 D_r}{4\pi} \frac{1}{R^2} = \left(\frac{\lambda}{4\pi R} \right)^2 D_r D_t \quad (34)$$

If the antennas are not ideal, we can include efficiency factors and rewrite the resulting Friis transmission equation in terms of both directivity and gain:

$$\frac{P_r}{P_t} = e_r e_t \left(\frac{\lambda}{4\pi R} \right)^2 D_r D_t = \left(\frac{\lambda}{4\pi R} \right)^2 G_r G_t. \quad (35)$$

One thing to note is that whenever irradiance (power density in W/m^2) is encountered in radiometric examples in an antenna class, it can be helpful to adopt the antenna notation W because of the confusion that arises when using E to represent irradiance in a class that discusses electric fields. Otherwise there is minimal difficulty teaching students to apply antenna and optical radiometry concepts consistently.

V. OTHER DERIVATIONS OF THE FRIIS EQUATION

Friis presented his transmission equation with an argument that resembles a simpler form of the antenna derivation summarized in section II but he left it in terms of antenna areas, so that the result is

$$\frac{P_r}{P_t} = \frac{A_r A_t}{\lambda^2 R^2}, \quad (36)$$

involving a product of antenna effective areas rather than a product of antenna gains or directivities. Hogg [17] also outlined derivations of the Friis transmission equation based on Fresnel zones, Gaussian beams, and modes (basically a plane-wave spectrum approach), all leading to the equation as given in eq. (36). At first glance a student might be confused with this form of the equation because the received-to-transmitted power ratio is inversely proportional to the wavelength squared, whereas as given in eqs. (19) and (35) it is directly proportional to the wavelength squared. The reason for the difference is that the latter two equations include a wavelength-squared term substituted for each antenna effective area. Therefore, in eq. (33), just before the effective-area substitution, the radiometric derivation yields the same result as given by Friis [1] and Hogg [17].

It is important to help students understand that the discussion of eq. (30) can be used to see that the Friis equation in the form of eqs. (19) and (35) effectively has a factor of λ^4 hidden within the product of directivities or gains, so that if an antenna size remains unchanged the transmit and receive beams both diffract into larger solid angles, with the net effect of providing a $1/\lambda^2$ reduction of received power. Conversely, the Friis equation in the form of eq. (36) directly predicts the appropriate diffraction behavior of lower received power at longer wavelengths when the antenna areas are unchanged.

VI. DISCUSSION AND CONCLUSION

In two subsequent offerings of a first-year graduate course on antennas, the Friis transmission equation was presented with both the dipole effective area and radiometric derivations discussed in this paper. Many of the students had seen the Friis equation previously in a senior-level course on communication systems or a junior-level course on electromagnetic theory. Before these two derivations were presented the students could recognize and use the equation, but were entirely unable to describe the physical meaning of the wavelength term. In questioning immediately following the presentation of the dipole derivation, none of the students accurately identified the wavelength term as arising from diffraction. However, immediately following a radiometric discussion similar to that described here, 100% of the students identified the source of the wavelength-squared term as diffraction and could explain, in basic terms at least, how the wavelength entered in and flowed through the derivation. Additionally, all of the students selected the radiometric presentation as the one they felt was most physically intuitive. Nevertheless, our antenna course continues to present both

derivations so that students learn to appreciate both the radiometric and antenna styles of thinking.

The Friis transmission equation is a sufficiently simple and practical tool that students generally enjoy using it as the basis of discussion of radiometric concepts. Students who have taken both antenna and photonics or optics classes have a particularly high level of appreciation for seeing the two fields brought together into a common discussion. There are several antenna concepts (e.g. directivity, throughput = λ^2 , etc.) that enhance optical radiometry, just as the radiometric approach allows an alternate development of the wavelength-squared term that helps students better understand its diffraction roots.

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