

EE 548

Ultra Wideband Radio:
System Capacity Limits and Link Budgets
Spring 2006



Limits in communications

- Noise
- Channel capacity (Shannon limit)
- Regulatory constraints

Result: fundamental limit on link capacity:
Throughput versus distance trade off

Degrees of freedom:

- Transmitter power
- Signaling method
- Receiver design



First physical limit: Noise

- Interference
- Thermal (white) noise

Noise Power in a bandwidth B : $N_0B = k_bTB$

$k_b = 1.38 \times 10^{-23} \text{ J / K}$ (Boltzman's constant)

T = temperature, degrees Kelvin (300 at room temp)

For 1 Hz BW, noise power = -174dBm at room temperature

In 1Mhz, noise power = -114dBm



Second physical limit: channel capacity

Shannon's law for
channel capacity C :

$$C = B \log_2(1 + S / N)$$

UWB: Increase B instead of S/N !

S/N needed for a given bandwidth and capacity:

$$\frac{S}{N} = 2^{C/B} - 1$$



Communication efficiency

Define communications efficiency, e_b , of a communication system as the ratio of the minimum energy per data bit E_b to the noise power density N_0 necessary to achieve a sufficiently small bit error rate:

$$e_b = 10 \log\left(\frac{E_b}{N_0}\right)$$

The smaller e_b , the better a system will perform in the presence of white noise



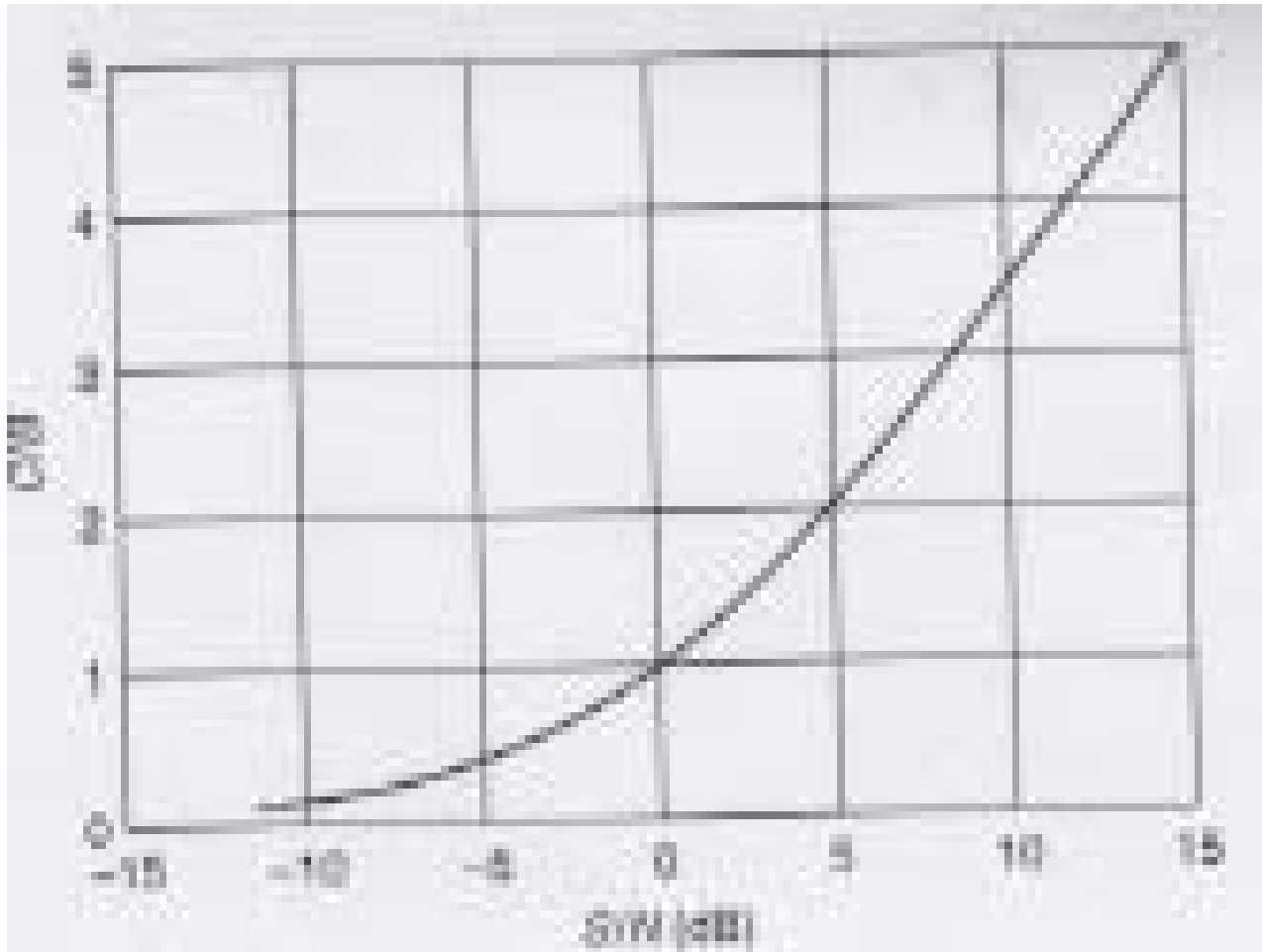
Example: POTS data line

- $B = 3\text{kHz}$
- $C = 56\text{kb/s}$
- $C/B = 56/3 = 18.67$
- $2^{18.67} = 416,128$
- $S/N \text{ (dB)} = 10\log(416,128) = 56\text{dB!}$
- IF S/N were 1, line would need $B = 56\text{kHz}$ to get the same capacity.

Trade off between S/N and B !!



Ideal channel performance



What about a UWB channel?

- $B = 10.6 - 3.1 = 7.5\text{GHz}$
- For S/N of 1 (0 dB), in an ideal channel
 $C = B = 7.5\text{Gb/s!!}$
- Hence at low S/N, can get high throughput by using wide bandwidth



Communication efficiency and ideal channel capacity

Want to use Shannon's law to find the smallest e_b (highest modulation efficiency) possible for error-free communication

$$\frac{S}{N} = \frac{(E_b R_b)}{(N_0 B)} = \frac{(E_b / N_0)}{(T_b B)}$$

where R_b = bit rate, bits/sec

$R_b = 1/T_b$, T_b = time to send one bit



Combine with Shannon's law:

$$\frac{S}{N} = 2^{C/B} - 1 = \frac{(E_b / N_0)}{(T_b B)}$$

Solve for C:

$$C = B \log_2 \left(\frac{E_b}{N_0 B T_b} + 1 \right)$$

Consider limit when B is large (UWB):

$$C = \frac{E_b}{(N_0 T_b \ln(2))}$$



Consider Shannon limit: information rate $1/T_b = C$

Then:

$$C = \frac{E_b C}{(N_0 \ln(2))}$$

and solving for $\frac{E_b}{N_0}$:

$$\frac{E_b}{N_0} = \ln(2), \text{ and}$$

$$e_b = 10 \log \left(\frac{E_b}{N_0} \right) = 10 \log(\ln(2)),$$

$$e_b = -1.59 \text{ dB}$$



Second limit

$$e_b = -1.59\text{dB}$$

This is the *best possible* communications efficiency in a bandwidth unconstrained channel with white noise!

However: Shannon theory does not specify how to achieve this limit. Various modulation techniques will yield less efficient results!



Modulation efficiencies relative to ideal ($e_b = -1.59\text{dB}$)

Modulation	Modulation efficiency relative to ideal, at BER = 10^{-3}
64 BOK (Bi-orthogonal keying modulation)	5.7
2-PAM and BPSK	8.4
PPM/OOK	11.4



Third factor: regulatory limits

- Maximum power emission level:
-41.3dBm/MHz
- Maximum BW: $10.6 - 3.1 = 7.5\text{GHz} = 7500\text{MHz}$
- EIRP (effective isotropic radiated power):
- $\text{EIRP} = -41.3 + 10\log(7500) = -2.55\text{dBm}$



Fundamental limit for UWB

- Noise
- Optimal modulation efficiency
- Allowed EIRP

Define UWB System Gain Limit, SG_{lim}

$$SG_{\text{lim}} = EIRP - N_1 - e_{b\text{min}}$$

This is the theoretical limit of system gain per bit rate



UWB System gain limit

- EIRP (max) = -2.55dBm
- $N_1 = -174\text{dBm/Hz}$
- $e_{\text{bmin}} = -1.59\text{dB}$

$$SG_{UWB} = 173\text{dB} / \text{bps}$$

Interpretation: There is 173dB of path loss available for a 1 bps UWB radio link



Fundamental limit and link budget

- Include path loss $P_L(d)$, $d =$ distance
- Include data bandwidth B
- Assume no antenna gains:

$$SNR_R = SG_{\text{lim}} + P_L(d) - 10 \log(B)$$



Fundamental UWB limit capacity

- Use SNR_R with Shannon channel capacity equation:

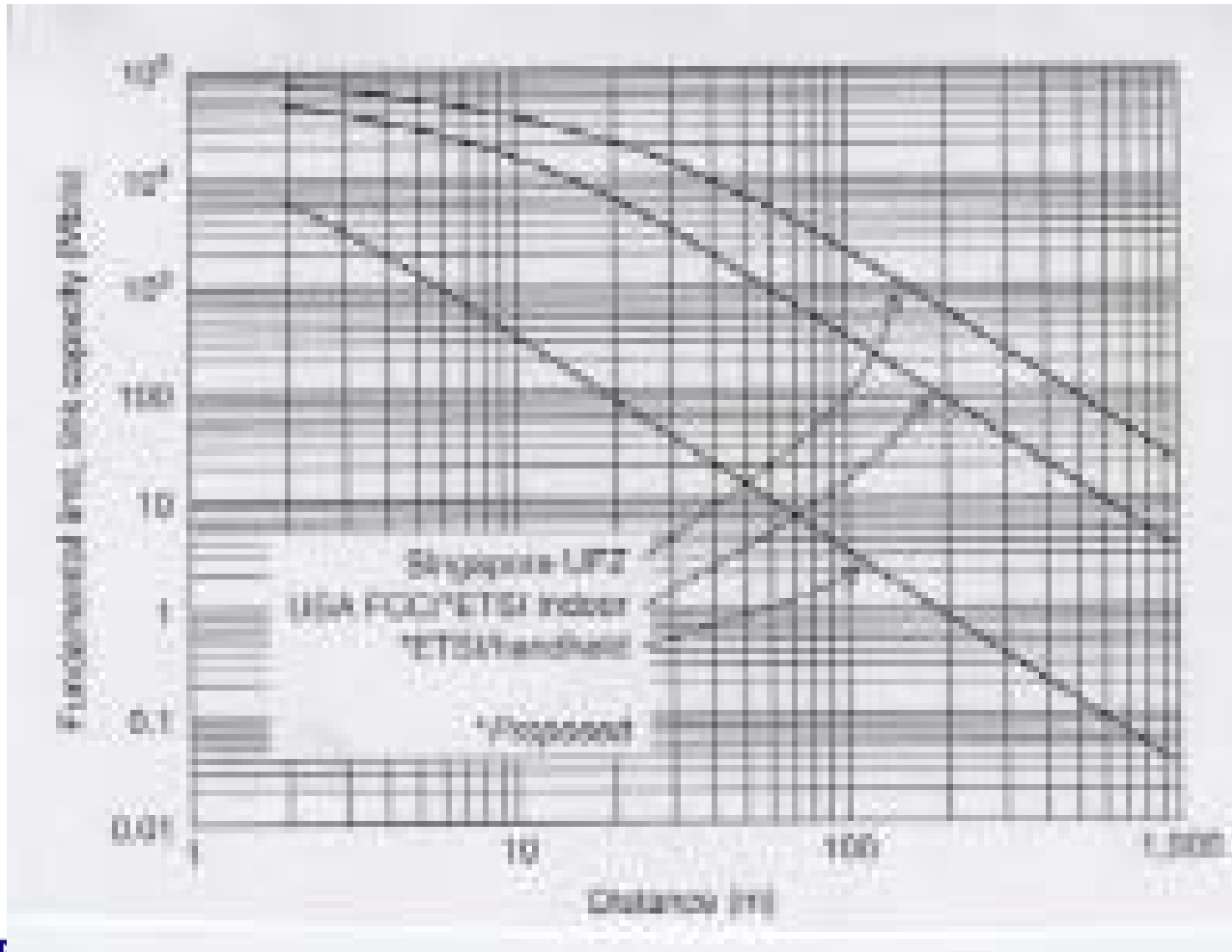
$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

$$C = B \log_2 \left(1 + 10^{SNR_R} \right)$$

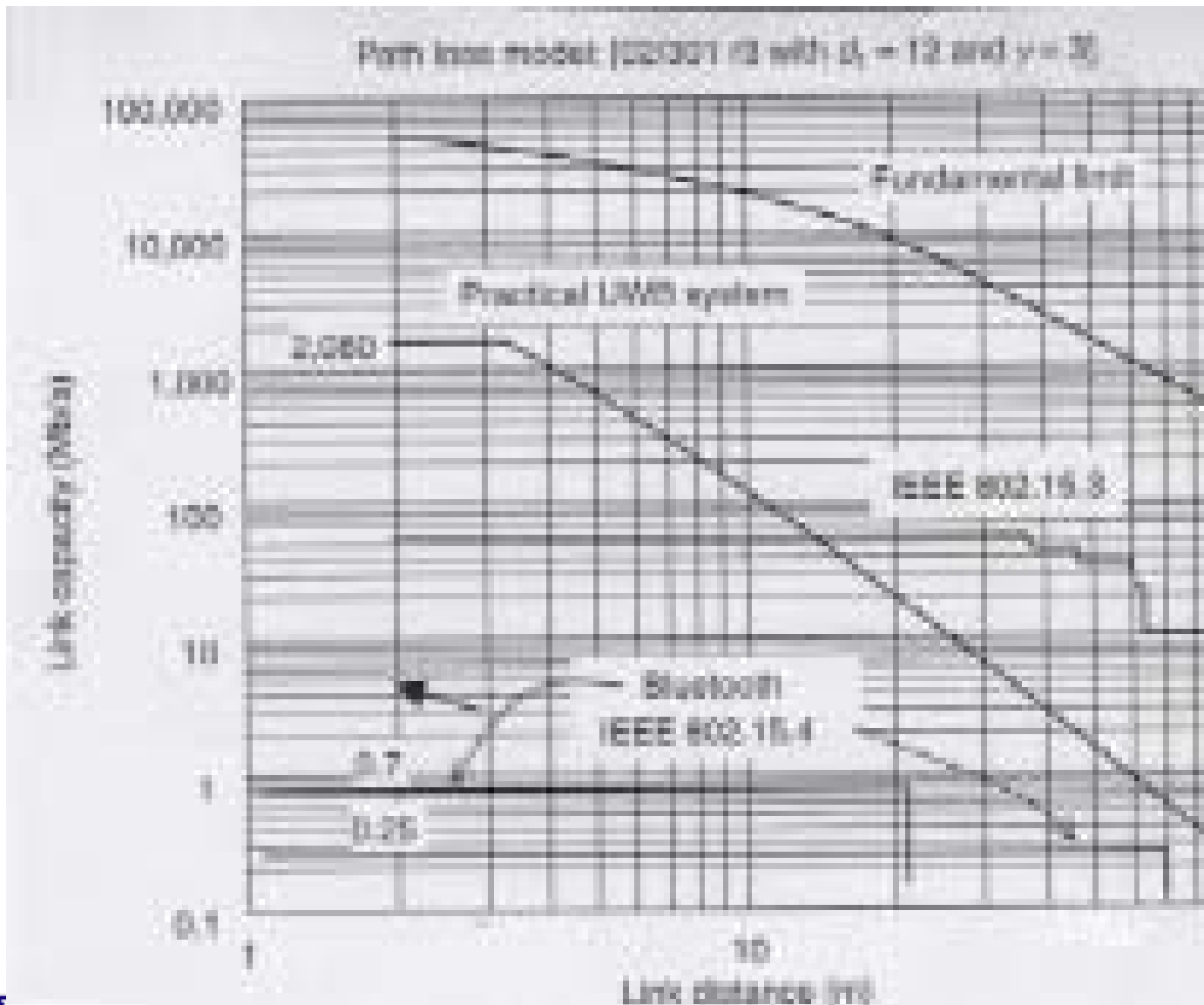
Since SNR_R is a function of distance due to path loss, this gives a trade off between channel rate C and distance!



UWB fundamental limit maximum capacity



UWB and narrowband systems compared



Link budget for an efficient high data rate UWB system

System parameter	Design choices	Value
System gain limit		173dB/bps
$-10\log(\text{link BW})$	Link BW = 515Mbps	-87.1dBbps
System losses (noise figure, implementation losses)		-10dB
Modulation efficiency	64-BOK at 10^{-3} BER	-5.7dB
$-10\log(\text{actual RF BW}/7.5\text{GHz})$	RF BW=1.4GHz	-7.3dB
Antenna gain/loss		0dBi
Available for path attenuation		62.9dB

