REU-Wireless Communications
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Decibels and Radio Links

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logarithms

- a **logarithm** (to base *b*) of a number *x* is the exponent *y* of the power *by* such that $x = b^y$.
- The base-*b* logarithm is usually written as $log_b(x)=y$
- Examples: use of base 10
  - $10^1 = 10$. Hence $\log_{10}(10)=1$
  - $10^0=1$. Hence $\log_{10}(1)=0$
  - $10^3 = 1000$. Hence $\log_{10}(1000)=3$
  - $10^{2.5} =316.23$. Hence $\log_{10}(316.23)=2.5$

- Convention: use notation $\log_{10}(x) = \log(x)$

bels

- bel: defined as a ratio of two numbers
- 1 bel = a factor of 10 increase relative to a reference level

$$\text{Power ratio in bels} = \log\left[\frac{P}{P_{ref}}\right]$$

Example: P =10, P_{ref} = 1

$$\text{Power ratio in bels} = \log\left[\frac{P}{P_{ref}}\right] = \log[10/1] = \log[10]$$

$$\text{Power ratio in bels} = 1.00$$

Decibels

Bel is a large unit, hence define decibel as one-tenth of a bel:

$$\text{Power ratio in decibels} = 10\log\left[\frac{P}{P_{ref}}\right]$$

1 decibel = 10 bels

- Examples
  - Suppose $P/P_{ref} = 10$
  - Ratio in decibels: 10log(10) = 10x1=10

- Convention: abbreviate “decibel” as “dB”

Decibels: more examples

Let X = P/P_{ref} and $Y = 10\log(X)$

- Suppose X = 2. Y = 10log(2) = 10 (0.301) = 3.01dB. Hence doubling a number increases it by 3 dB!

- Suppose X=10.
Y=10log(10) =10 (1) = 10dB
- Suppose X=100.
Y=10log(100) = 10 (2) = 20dB
- Suppose X = 0.1
Y=10log(0.1) =10 (-1) = -10dB

$$\text{Power ratio in decibels} = 10\log\left[\frac{P}{P_{ref}}\right]$$

Decibels: more examples

Suppose take the log of a number raised to a power:

$Y = 10\log(X^N)$

Then $Y = N[10\log(X)]$

For $N = 2$:

$Y = 20\log(X)$

$$\text{Power ratio in decibels } P = 10\log\left[\frac{V^2}{V^2_{ref}}\right] = 10\log\left[\left(\frac{V}{V_{ref}}\right)^2\right]$$

$$P = 20\log\left[\frac{V}{V_{ref}}\right]$$

Power and voltage

- Note that power is proportional to V^2

$$P = \left[\frac{V^2}{R}\right] \text{(Ohm's law)}$$

•Then in decibels (dB):

$$\text{Power ratio in decibels P} = 10\log\left[\frac{V^2}{V^2{}_{ref}}\right] = 10\log\left[\left(\frac{V}{V_{ref}}\right)^2\right]$$

$$P = 20\log\left[\frac{V}{V_{ref}}\right]$$

Power and voltage

- Note that power is proportional to V^2
- Suppose the voltage is doubled
- V/V_{ref} goes up by a factor of 2, and
- In dB, V/V_{ref} increases by 3 dB
- P/P_{ref} goes up by a factor of 2^2, and
- In dB, P/P_{ref} increases by 6 dB

Power relative to a reference level

- Convenient to reference power levels to watts (W) or miliwatts (mW):

$$1W = 1000mW$$

- Then in decibels (dB):

$$0dBW = 30dBm$$

$$1\mu W = 10^{-3} mW = 10^{-6} W$$

$$0\mu W(dB) = -30dBm = -60dBW$$

Why use dB?

- Logs make multiplication and division easy:
- $\log(XY) = \log(X) + \log(Y)$
- $\text{Log}(X/Y) = \log(X) - \log(Y)$
- Logs make it easier to compare numbers that are vastly different
  - Suppose $X = 1$ and $Y = 1{,}000{,}000$
  - $Y/X=1{,}000{,}000$ (big number)
  - In dB, $10\log(Y/X) = 60$ (not such a big number)
- Very useful when graphing numbers that have a wide range: use log scales on X and/or Y axes